Basic Concepts

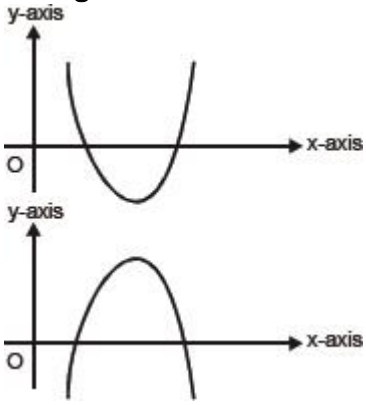
- Zeroes of a polynomial. k is said to be zero of a polynomial p(x) if p(k) = 0
- Graph of polynomial.

(i) Graph of a linear polynomial ax + b is a straight line.

(ii) Graph of a quadratic polynomial $p(x) = ax^2 + bx + c$ is a parabola open upwards like ∪, if $a > 0$.

(iii) Graph of a quadratic polynomial $p(x) = ax^2 + bx + c$ is a parabola open downwards like ∩, if $a > 0$.

(iv) In general a polynomial p(x) of degree n crosses the x-axis at atmost n points.



- Relationship between the zeroes and the coefficients of a Polynomial.

(i) If α, β are zeroes / roots of $p(x) = ax^2 + bx + c$, then

$$\text{Sum of roots } \alpha+\beta = \frac{-b}{a} \Rightarrow \alpha+\beta = \frac{-(\text{coefficient of } x)}{\text{coefficient of } x^2}$$

$$\text{Product of roots} = \alpha\beta = \frac{c}{a} \Rightarrow \alpha\beta = \frac{\text{constant term}}{\text{coefficient of } x^2}$$

- If α, β are roots of a quadratic polynomial p(x), then $p(x) = x^2 - (\alpha + \beta)x + \alpha\beta$

$\Rightarrow p(x) = x^2 - (\text{sum of roots})\, x + \text{product of roots}$

Example: Find the zeroes of the quadratic polynomial and verify the relationship between the zeroes and coefficient of polynomial $p(x) = x^2 + 7x + 12$.

Sol. $p(x) = x^2 + 7x + 12$

$\Rightarrow p(x) = (x + 3)(x + 4)$

$\therefore p(x) = 0$ if $x + 3 = 0$ or $x + 4 = 0$

$\Rightarrow x = -3$ or $x = -4$

$\therefore -3$ and -4 are zeros of the p(x).

Now,

$$\text{Sum of the zeroes} = -3 + (-4) = -7 = \frac{-7}{1} = \frac{-(\text{coefficient of } x)}{\text{coefficient of } x^2}$$

$$\text{Product of the zeroes} = (-3) \times (-4) = 12 = \frac{12}{1} = \frac{\text{constant term}}{\text{coefficient of } x^2}$$

2. Find the zeroes of $4x^2 - 7$ and verify the relationship between the zeroes and its coefficients

Sol. Let $p(x) = 4x^2 - 7$

Here coefficient of $x^2 = 4$,

Coefficient of $x = 0$ and constant term $= -7$.

Now $p(x) = 4x2 - 7 = (2x - \sqrt{7})(2x + \sqrt{7})$

$\therefore p(x) = 0$, if $2x - \sqrt{7} = 0$ or $2x + \sqrt{7} = 0$

$$\Rightarrow x = \frac{\sqrt{7}}{2} \text{ or } x = \frac{-\sqrt{7}}{2}$$

$$\therefore \frac{\sqrt{7}}{2} \text{ and } \frac{-\sqrt{7}}{2} \text{ are zeroes of } p(x).$$

Now,

$$\text{Sum of zeroes} = \frac{\sqrt{7}}{2} + \left(\frac{-\sqrt{7}}{2}\right) = 0 = \frac{0}{4} = \frac{-(\text{coefficient of } x)}{\text{coefficient of } x^2}$$

$$\text{Product of zeroes} = \frac{\sqrt{7}}{2} \times \frac{-\sqrt{7}}{2} = \frac{-7}{4} = \frac{\text{constant term}}{\text{coefficient of } x^2}$$

3. Find a quadratic polynomial whose zeroes are $5 + \sqrt{2}$ and $5 - \sqrt{2}$.

Sol. Let α, β are zeroes of quadratic polynomial $p(x)$.

$\therefore p(x) = x2 - (\alpha + \beta)x + \alpha\beta$

Here, $\alpha = 5 + \sqrt{2}$, $\beta = 5 - \sqrt{2}$

$\therefore \alpha + \beta = 5 + \sqrt{2} + 5 - \sqrt{2} = 10$

and $\alpha\beta = (5 + \sqrt{2})(5 - \sqrt{2})$

$= 25 - 4 = 21$

$\therefore p(x) = x^2 - 10x + 21$